# AN ALGORITHM FOR REAR-END COLLISION AVOIDANCE WARNING SYSTEMS

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Paper Number 310

## ABSTRACT

The National Highway Traffic Safety Administration (NHTSA), supported by The Johns Hopkins University Applied Physics Laboratory (JHU/APL), has developed an algorithm for use with rear-end collision avoidance systems that alerts drivers to potentially dangerous driving situations and the need to take evasive action. This algorithm is to be integrated into a General Motors (GM) developed collision warning system for use during the Automotive Collision Avoidance System (ACAS) Field Operational Test (FOT). The NHTSA algorithm uses the host vehicle velocity and acceleration along with the collision warning system-supplied values for range, range rate, and relative acceleration of the lead vehicle to calculate a miss-distance between the host and lead vehicles at 0.1-second intervals. The miss-distance is the closest distance that occurs between the two vehicles if the driver of the host vehicle were to initiate braking after a delay time at a designated host vehicle maximum braking capability. This calculated distance is compared to a miss-distance threshold and if it is less, a warning is provided to the driver. The algorithm accounts for a driver sensitivity setting and includes a look-ahead calculation to determine if the threshold would be passed before the next time interval. The performance of the algorithm has been examined against designated operational scenarios. These scenarios include cases of a constant speed host vehicle encountering a stopped lead vehicle, a constant speed host vehicle encountering a constant but slower speed lead vehicle, and a constant speed host vehicle encountering a lead vehicle braking from the same initial speed.

## INTRODUCTION

NHTSA has sponsored work in rear-end collision avoidance since 1992. Key results of this work were presented at the NHTSA sponsored JHU/APL Rear-End Collision Avoidance Symposium (Reference 1). Along with sponsoring work in the rear-end collision avoidance area, NHTSA developed an experimentally based rear-end collision warning algorithm (Reference 2) and sponsored analysis of the warning algorithm by Mitretek using field operational test data (Reference 3) collected during the Intelligent Cruise Control Field Operational Test.

The NHTSA Alert Algorithm is an extension of this previous work designed for use with a GM developed collision warning system during the ACAS FOT. The ACAS FOT will be conducted as part of a five-year cooperative research agreement between NHTSA and GM to develop and field test a limited fleet of vehicles equipped with an adaptive cruise control system and a prototype rear-end collision warning system. A key part of the collision warning system is the algorithm that issues drivers warnings to alert them of potentially dangerous driving situations and the need to take evasive action. As part of this research agreement, NHTSA will provide an alert algorithm that will be used along with GM developed alert algorithms during the FOT.

The objective of the NHTSA Alert Algorithm is to issue collision alert warnings that allow the driver of the host vehicle to stop or approach no closer than a designated miss-distance threshold behind a lead vehicle. The decision to issue an alert warning is made every 100 milliseconds upon parameter update by the collision warning system. The algorithm computes the projected miss-distance at each time interval based on an assumed constant host vehicle deceleration and estimates of host and lead vehicle speed and acceleration and the range and range rate between the vehicles. This projected miss-distance is then compared with a miss-distance threshold to determine if a warning should be issued.

The NHTSA Alert Algorithm provides four alert levels. The lowest level corresponds to "no warning." The highest alert level is an "imminent collision" warning, for which the miss-distance is calculated using an assumed host vehicle maximum braking capability. A driver-set 3-level warning sensitivity parameter scales the maximum braking capability used by the algorithm. Two intermediate alert levels provide early warning of collisions based on the host vehicle braking at reduced levels.

The miss-distance threshold has a fixed component and a variable component that is a function of the host vehicle speed. The variable component is based on a "look ahead" of one time interval to assure that the fixed component would not be violated before the next set of parameters is received. Warnings are issued when the calculated miss-distance is less than the threshold miss-distance in two of the last three time intervals for a warning level. A warning may be suppressed when the driver has recently braked or been warned, or the host vehicle speed is below a threshold level. Once a warning is issued, an alert level is normally maintained for a minimum of one second or while the range between the two vehicles is closing, unless a higher alert level is issued. An alert may be cleared or reduced if a new target replaces the lead vehicle.

## ALGORITHM DEVELOPMENT
### Background

The following assumptions were implicit in the development of the algorithm:

   a. The current input parameter values represent the best available estimates and are stable and accurate.
   b. The lead vehicle will maintain its current deceleration.
   c. The host vehicle will maintain its current acceleration for a fixed reaction time and then decelerate at a constant prescribed level.
   d. The system is inhibited from providing warnings when the host vehicle velocity is $\leq 5$ m/s.

The input parameters required by the NHTSA Alert Algorithm at each time interval are:

   a. Host Vehicle Velocity ($V_H$) – supplied by the collision warning system
   b. Host Vehicle Acceleration ($A_H$) – supplied by the collision warning system
   c. Lead Vehicle Velocity ($V_L$) – calculated as the sum of $V_H$ and the range rate supplied by the collision warning system
   d. Lead Vehicle Acceleration ($A_L$) – calculated as the sum of $A_H$ and the relative acceleration $A_R$ supplied by the collision warning system
   e. Range (R) – supplied by the collision warning system
   f. Range Rate (RR) – supplied by the collision warning system

Additional variables or constants used by the NHTSA Alert Algorithm include:

   a. Reaction Time ($T_R$) – This time, which accounts for both the driver reaction time and the system delay time that occur prior to full application of vehicle braking after a warning is issued, is currently set to 1.5 seconds.
   b. Assumed Host Vehicle Maximum Braking Capability ($A_{Hmax}$) – This value is currently set to -0.6 g. It should be noted that the $A_{Hmax}$ used in the miss-distance equations may be reduced by the driver-set Warning Sensitivity and may also be further reduced to determine the miss-distances for the intermediate alert levels.
   c. Warning Sensitivity – This variable may be set to one of three levels by the driver; it is expected that the lowest sensitivity level, which implies a desire for shorter headways between the host and lead vehicles, would be chosen by a more aggressive driver.
   d. Brake ON/OFF – This variable indicates whether the brake has been applied by the driver (ON) or released (OFF).

As noted above $A_{Hmax}$ is scaled prior to use in the miss-distance equations by the Warning Sensitivity to provide earlier warning to those desiring it. The current values of $A_{Hmax}$ corresponding to the three warning sensitivity levels are:

   a. Low Sensitivity (Desire for shorter headways): $A_{Hmax}$ = -0.6 g
   b. Medium Sensitivity: $A_{Hmax}$ = -0.5 g
   c. High Sensitivity (Desire for longer headways): $A_{Hmax}$ = -0.4 g

The ACAS FOT collision warning system provides input parameters for multiple targets. These include the Closest In-Path Moving Vehicle (CIPV) and the Closest In-Path Stationary Object (CIPS). The NHTSA Alert Algorithm processes the CIPV and CIPS targets in parallel when present, effectively running duplicate code processes for each type of target.

It should be noted that the NHTSA Alert Algorithm assumes that the targets provided to it are actual targets in the path of the host vehicle; there is no attempt made by the algorithm to determine if the targets presented to it are valid. Also, it was implicit in the development of the NHTSA Alert Algorithm that the input parameter values are stable and accurate. If the input velocity, acceleration, or range parameters are noisy, unstable, or unreliable, filtering

may need to be applied to the input parameters or the NHTSA Alert Algorithm may require modification.

**Miss-Distance Equations**

There are two basic cases that must be considered in the calculation of miss-distance – either the lead vehicle stops prior to the host vehicle or the host vehicle stops while the lead vehicle is still in motion. The calculation of miss-distance occurs differently for the two cases. To determine the case that applies, the time for the lead vehicle to stop ($T_{LS}$) and the time for the host vehicle to stop ($T_{HS}$) are calculated as follows and compared:

$$T_{LS} = -V_L/A_L \qquad (1.)$$
and
$$T_{HS} = T_R - (V_H + A_H T_R) / A_{Hmax} \qquad (2.)$$

When the lead vehicle comes to a stop first, the minimum miss-distance ($D_{miss}$) between the vehicles will occur when the host vehicle comes to a stop. This case can be divided into three segments depending on the relationship between $T_{LS}$ and $T_R$. When $T_{LS} \geq T_R$ these segments are: (1) from the present time through the host vehicle reaction time (t = 0 to $T_R$), (2) from the host vehicle reaction time until the lead vehicle stops (t = $T_R$ to $T_{LS}$), and (3) the time from when lead vehicle stops until the host vehicle stops (t = $T_{LS}$ to $T_{HS}$). When $T_{LS} < T_R$ these segments are: (1) from the present time until the lead vehicle stops (t = 0 to $T_{LS}$), (2) the time from when lead vehicle stops through the host vehicle reaction time (t = $T_{LS}$ to $T_R$), and (3) the time from the host vehicle reaction time until the host vehicle stops (t = $T_R$ to $T_{HS}$). This miss-distance is calculated as:

$$D_{miss} = R + \Delta R1 + \Delta R2 + \Delta R3 \qquad (3.)$$

Where for the case $T_{LS} \geq T_R$:

$$\Delta R1 = (RR)T_R + \tfrac{1}{2}(A_L - A_H)(T_R)^2 \qquad (4.)$$

$$\Delta R2 = (RR + (A_L - A_H)T_R)(T_{LS} - T_R) + \tfrac{1}{2}(A_L - A_{Hmax})(T_{LS} - T_R)^2 \qquad (5.)$$

$$\Delta R3 = (RR + (A_L - A_H)T_R + (A_L - A_{Hmax})(T_{LS} - T_R))(T_{HS} - T_{LS}) + \tfrac{1}{2}(0 - A_{Hmax})(T_{HS} - T_{LS})^2 \qquad (6.)$$

Equations 3 through 6 simplify to:

$$D_{miss} = R + \tfrac{1}{2}(A_H - A_{Hmax})(T_R)^2 - \tfrac{1}{2}A_L(T_{LS})^2 - (A_H - A_{Hmax})T_R T_{HS} + (RR)T_{HS} + A_L T_{HS}T_{LS} - \tfrac{1}{2}A_{Hmax}(T_{HS})^2 \qquad (7.)$$

When $T_{LS} < T_R$ Equations 4 through 6 are set up similarly and also simplify to Equation 7.

When the host vehicle comes to a stop first, the minimum miss-distance between the vehicles will occur when the range rate equals zero. This case can be divided into two segments: (1) from the present time through the host vehicle reaction time (t = 0 to $T_R$), and (2) from the host vehicle reaction time until the minimum miss-distance occurs (t = $T_R$ to $T_M$). The range rate at time t = $T_M$ is calculated as:

$$RR_{T_M} = 0 =$$

$$(RR + (A_L - A_H)T_R) + (A_L - A_{Hmax})(T_M - T_R) \qquad (8.)$$

Which yields:

$$T_M = ((RR + (A_L - A_H)T_R) / (A_{Hmax} - A_L)) + T_R \qquad (9.)$$

The miss-distance is calculated as:

$$D_{miss} = R + \Delta R1 + \Delta R2 \qquad (10.)$$

Where:

$$\Delta R1 = (RR)T_R + \tfrac{1}{2}(A_L - A_H)(T_R)^2 \qquad (11.)$$

$$\Delta R2 = (RR + (A_L - A_H)T_R)(T_M - T_R) + \tfrac{1}{2}(A_L - A_{Hmax})(T_M - T_R)^2 \qquad (12.)$$

Equations 10 through 12 simplify to:

$$D_{miss} = R + (RR)T_M + \tfrac{1}{2}(A_L - A_{Hmax})(T_M)^2 - (A_H - A_{Hmax})T_M T_R + \tfrac{1}{2}(A_H - A_{Hmax})(T_R)^2 \qquad (13.)$$

**Multiple Alert Levels**

As stated previously, the NHTSA Alert Algorithm provides four alert levels, a "no warning" level and three levels of warning. The highest of the three warning levels is an "imminent collision" warning, for which the miss-distance is calculated using an assumed host vehicle maximum braking capability, while the two intermediate alert levels provide early warning of collisions based on the host vehicle braking at reduced levels. The algorithm calculates the three miss-distances in parallel.

As previously noted the maximum braking capability is scaled prior to the calculation of miss-distances by the driver-set 3-level warning sensitivity parameter. The "maximum brake" miss-distance is based on

using this warning sensitivity scaled $A_{Hmax}$. A "medium brake" miss-distance is based on using a value of 0.75 of the warning sensitivity scaled $A_{Hmax}$. While, a "minimum brake" miss-distance is based on using a value of 0.50 of the warning sensitivity scaled $A_{Hmax}$.

**Miss-Distance Threshold**

The miss-distance threshold ($D_{thresh}$) to which the projected miss-distance ($D_{miss}$) for each target vehicle at each time interval is compared, has a fixed component of 2 meters and a variable component that is a function of the host vehicle speed. The variable component is based on a "look ahead" of one time interval to assure that the fixed component would not be violated before the next set of parameters is received. This look ahead is computed as the host vehicle velocity ($V_H$) times the time interval (0.1 sec). Thus the miss-distance threshold is calculated as:

$$D_{thresh} = 2 \text{ m} + (V_H)(0.1 \text{ s}) \qquad (14.)$$

**Alert Generation**

While the decision to issue an alert warning is nominally based on a miss-distance threshold being exceeded as discussed above, the warning is not issued unless additional conditions are satisfied. The first condition is that the miss-distance threshold must be exceeded in two of the last three time intervals. If this condition is passed the warning is issued unless it is suppressed by one of three conditions – "Just Braked," "Just Warned," or if the host vehicle velocity ($V_H$) is ≤ 5 m/s.

The "Just Braked" suppression condition is meant to inhibit the issuing of alerts to an attentive driver. It goes into effect and continues for 2 seconds after the "Brake OFF" signal is received from the collision warning system. The "Just Warned" suppression condition is also meant to inhibit the issuing of alerts to an attentive driver. It inhibits the issuing of new alerts for the current target vehicle for 5 seconds after a previous alert is cleared; it does not prevent issuing alerts to new target vehicles.

It should be noted that only the highest warning level applicable to a target is issued, and that once an alert is issued, it is kept on for a minimum of one second unless a higher alert level is required or the target vehicle changes. After one second, the alert level may go to a lower level or be cleared, if the Range Rate (RR) is > -1 m/s.

**Output Parameters**

The NHTSA Alert Algorithm provides three types of output data. These are the Alert Level, Algorithm Status Word, and Diagnostic Data. The Alert Level, which represents the current state of warning, is the primary output of the algorithm. This four-state output is used by the Collision Warning System Driver Vehicle Interface (DVI) to alert the driver. As noted previously, the NHTSA Alert Algorithm processes the CIPV and CIPS targets in parallel and thus produces an alert level for each target type. The overall Alert Level for the NHTSA Alert Algorithm is the highest alert level produced for either the CIPV or CIPS target.

The Algorithm Status Word provides additional information on the status of operation of the NHTSA Alert Algorithm. It may have one of four values, indicating that the system is in a normal operational status, that the system is inhibited and alerts are not being issued, that the system capabilities are being exceeded, or that there is a system fault (e.g., the collision warning processor being in a non-operational state). Currently, the system is inhibited if the host vehicle velocity ($V_H$) is ≤ 5 m/s. If the status word indicates that the system's capabilities are being exceeded, this is indicated to the driver through the DVI. Alerts continue to be issued during this system limitation condition. The Diagnostic Data outputs are for use in determining the performance of the algorithm and analyzing its operation. They are not used in the operation of the collision avoidance system.

**ALGORITHM PERFORMANCE**

During the ACAS FOT the NHTSA Alert Algorithm will be evaluated using a collision-warning simulator and in-vehicle testing. In an attempt to understand the performance of the NHTSA Alert Algorithm prior to these tests, the operation of the algorithm is currently being examined for a series of operational scenarios under the assumption of perfect input data.

The base scenarios for this initial examination of performance include:

a. A constant speed host vehicle encountering a stopped lead vehicle
b. A constant speed host vehicle encountering a constant but slower speed lead vehicle
c. A constant speed host vehicle encountering a lead vehicle braking from the same initial speed.

The case of a constant speed host vehicle encountering a stopped lead vehicle has been estimated using General Estimates System (GES) data from 1992-1996 (Reference 4) to be about 33% of rear-end crashes. Table 1 provides the initial acquisition range required by the algorithm to provide an "imminent collision" warning for this scenario as a function of host vehicle velocity and maximum braking capability for a 1.5 second reaction time. As expected, the required acquisition range increases with vehicle velocity and decreases as the braking capability is increased. It should be noted that the acquisition ranges required for the higher vehicle velocities may not be available due to radar system limitations or the operational situation (e.g., a curved road). This does not indicate a failure of the algorithm, but rather an operational limitation that needs to be conveyed to the user of the system.

Figure 1 shows the timing of each of the three alert levels for the NHTSA Alert Algorithm when a constant speed host vehicle encounters a stopped lead vehicle at an initial range of 125 m with an assumed maximum braking capability of -0.6 g. As the speed of the host vehicle increases, the alerts are issued earlier and the time period between the alerts increases. The convergence of the curves at the highest shown velocities is indicative of the system's capabilities being exceeded and the alerts being issued upon target acquisition.

**Table 1.**
**Initial Acquisition Range Required By Alert Algorithm for Imminent Collision Warning of a Stopped Vehicle**

| Host Vehicle Velocity (kph) | Required Initial Range (m) Assumed Maximum Braking Capability | | |
|---|---|---|---|
| | -0.6 g | -0.5 g | -0.4 g |
| 24 | 17 | 18 | 19 |
| 40 | 31 | 33 | 36 |
| 56 | 48 | 53 | 58 |
| 72 | 69 | 75 | 86 |
| 89 | 94 | 105 | 120 |
| 105 | 122 | 136 | 158 |

The case of a constant speed host vehicle encountering a constant but slower speed lead vehicle has been estimated using GES data from 1992-1996 to be about 15% of rear-end crashes. Table 2 provides the initial acquisition range required by the algorithm to provide an "imminent collision" warning for this scenario as a function of the difference in velocity between the host and lead vehicles and maximum braking capability for a 1.5 second reaction time.
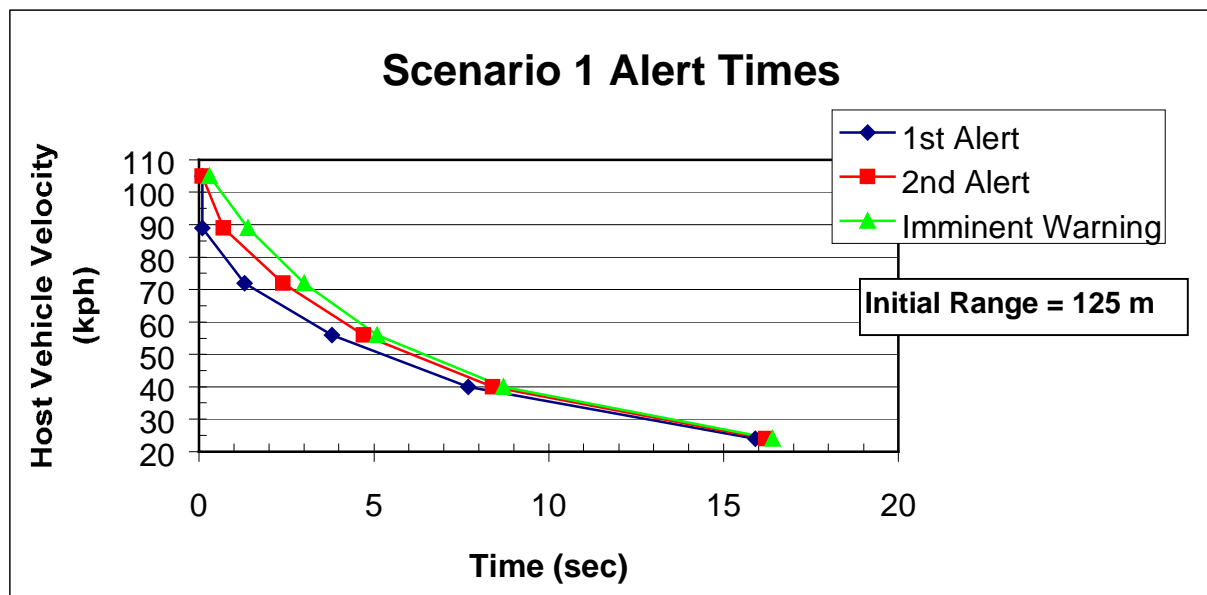


**Figure 1. The performance of the NHTSA Alert Algorithm, shown as the time of the three alert levels, when a constant speed host vehicle encounters a stopped lead vehicle at an initial range of 125 m with an assumed maximum braking capability of –0.6 g.**

**Table 2.**
**Initial Acquisition Range Required By Alert Algorithm for Imminent Collision Warning of a Slower Speed Lead Vehicle**

| Host-Lead Delta Velocity (kph) | Required Initial Range (m) | | |
|---|---|---|---|
| | Assumed Maximum Braking Capability | | |
| | -0.6 g | -0.5 g | -0.4 g |
| 20 | 16 | 17 | 18 |
| 30 | 24 | 25 | 27 |
| 40 | 32 | 35 | 38 |
| 50 | 43 | 46 | 51 |
| 60 | 54 | 56 | 66 |
| 70 | 67 | 73 | 83 |
| 80 | 81 | 89 | 102 |

This table was computed using a host vehicle velocity of 100 kph. Due to the variable component of the miss-distance threshold, there would be slight differences in the table for other host vehicle velocities. The required acquisition range increases with the difference in velocity between the host and lead vehicles and decreases as the braking capability is increased. The acquisition ranges required for this scenario are within the expected capability of the radar system, but again may not be available due to the operational situation at the higher velocities.

Figure 2 shows the timing of each of the three alert levels for the NHTSA Alert Algorithm when a constant speed host vehicle encounters a constant but slower speed lead vehicle at an initial range of 125 m with an assumed host vehicle maximum braking capability of -0.6 g. As the difference in velocity between the host and lead vehicles increases, the alerts are issued earlier, and the time period between the alerts increases.

The case of a constant speed host vehicle encountering a lead vehicle braking from the same initial speed has been estimated using GES data from 1992-1996 to be about 38% of rear-end crashes. Table 3 provides the initial acquisition range required by the algorithm to provide an "imminent collision" warning for this scenario as a function of the initial velocity and host vehicle maximum braking capability with the lead vehicle braking at –0.4g. The required acquisition ranges for this scenario are within the expected capability of the radar system
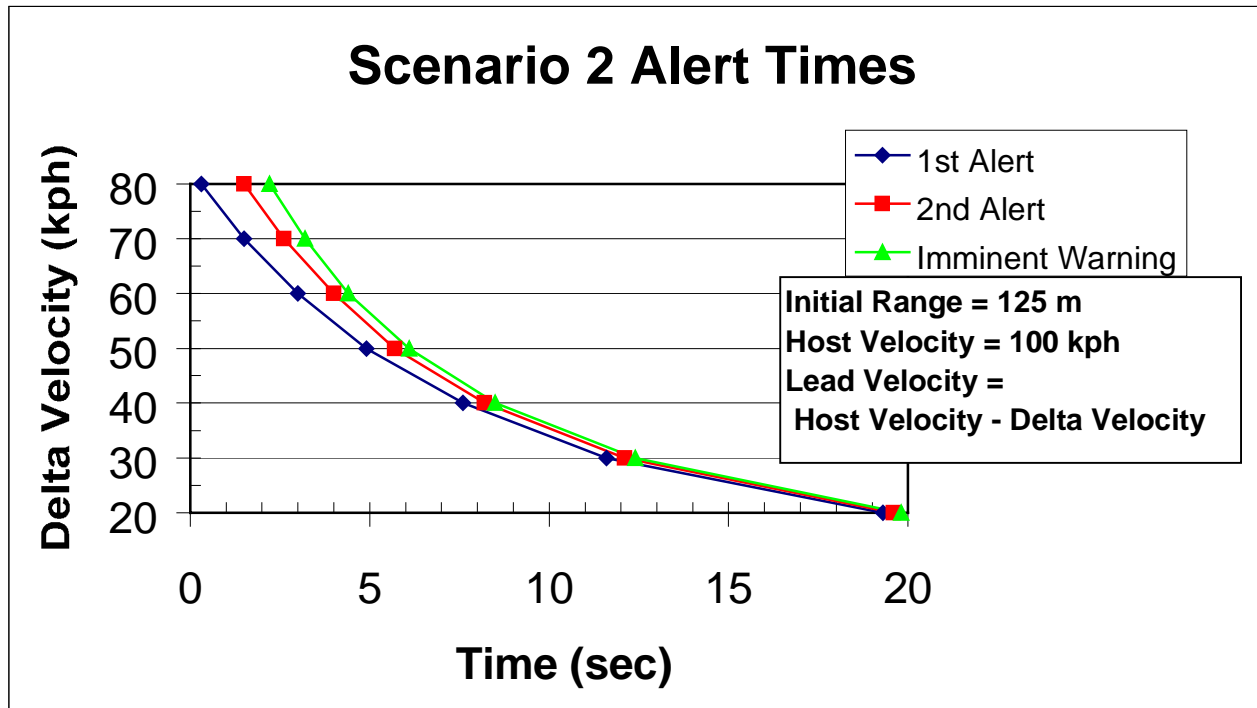


**Figure 2. The performance of the NHTSA Alert Algorithm, shown as the time of the three alert levels, when a constant speed host vehicle encounters a slower constant speed lead vehicle at an initial range of 125 m with an assumed maximum braking capability of –0.6 g.**

and should not be seriously affected by the operational situation.

**Table 3.**
**Initial Acquisition Range Required By Alert Algorithm For Imminent Collision Warning of a Vehicle Braking From Same Initial Speed**

| Vehicle Velocity (kph) | Required Initial Range (m) | | |
|---|---|---|---|
| | Assumed Maximum Braking Capability | | |
| | -0.6 g | -0.5 g | -0.4 g |
| 24 | 11 | 12 | 13 |
| 40 | 15 | 17 | 20 |
| 56 | 17 | 21 | 27 |
| 72 | 18 | 24 | 35 |
| 89 | 18 | 26 | 42 |
| 105 | 19 | 27 | 49 |

Figure 3 shows the timing of each of the three alert levels for the NHTSA Alert Algorithm when a constant speed host vehicle encounters a lead vehicle braking at –0.4g from the same initial speed at an initial range of 125 m with an assumed host vehicle maximum braking capability of -0.6 g. As expected, the alerts are issued earlier, and the time period between the alerts increases when the host vehicle velocity increases.

**SUMMARY**

This paper has reported the development by NHTSA of an algorithm that generates warnings for the purpose of alerting drivers to potential rear-end collision situations allowing them to take evasive action. The assumptions upon which the algorithm is based, the input parameters used by the algorithm, and the miss-distance equations used to determine the need for an alert are reviewed prior to discussing the performance of the algorithm under operational scenarios that are estimated to cover about 86% of rear-end crashes.

The alert algorithm is currently being incorporated into the ACAS collision warning system and its performance will be evaluated using a collision-warning simulator and in-vehicle testing. Possible concerns with its operation at high host vehicle velocities in some scenarios will be addressed and implementation issues investigated.
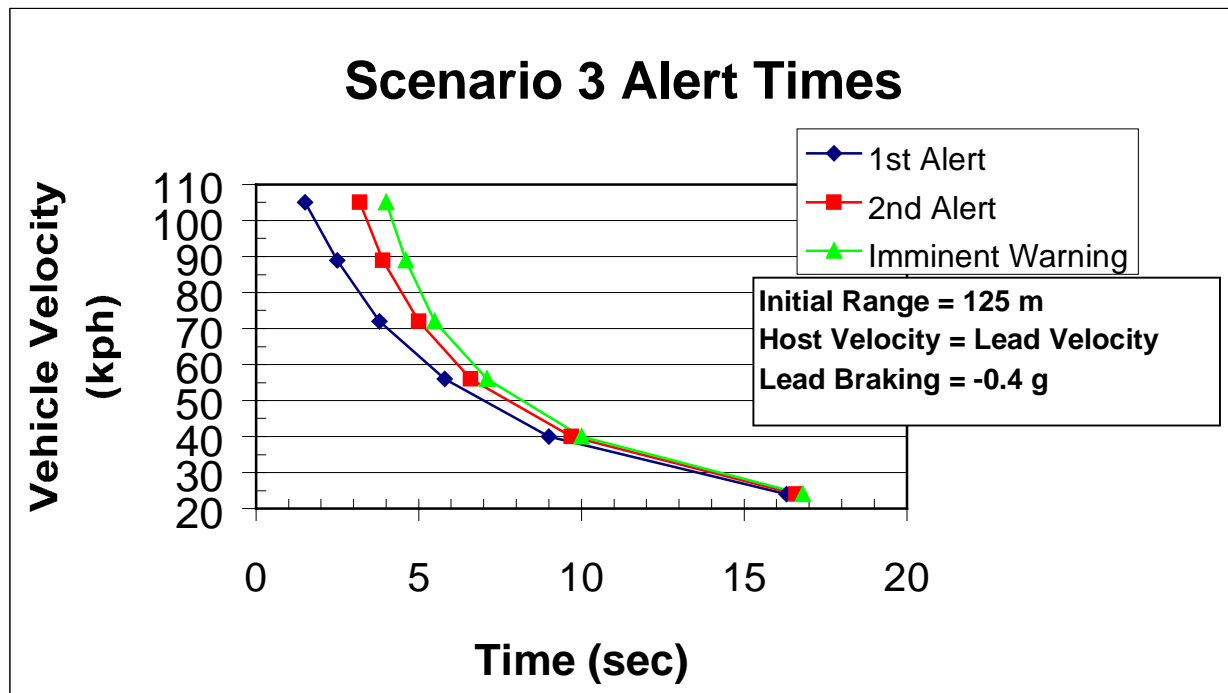


**Figure 3. The performance of the NHTSA Alert Algorithm, shown as the time of the three alert levels, when a constant speed host vehicle encounters a lead vehicle braking from the same initial speed at an initial range of 125 m. The lead vehicle is braking at –0.4g and the assumed host vehicle maximum braking capability is –0.6 g.**

Possible implementation issues include noisy values for host vehicle, and relative acceleration that may result in a need to filter data inputs beyond that provided by the collision warning system or to modify the algorithm. In addition, the scaling values used for modifying host vehicle maximum braking capability to account for the driver-set warning sensitivity or to generate multiple alert levels will be investigated to improve the operation of the algorithm. Another area to be investigated is the prevention of nuisance alerts to drivers, including the use of suppression conditions meant to inhibit the issuing of alerts to attentive drivers.

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